MENIIT

NEET | IIT-JEE | FOUNDATION

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JEE MAIN-2021

COMPUTER BASED TEST (CBT)

DATE : 18-03-2021 (EVENING SHIFT) | TIME : (3.00 pm to 6.00 pm)

Duration 3 Hours | Max. Marks : 300

QUESTION & SOLUTIONS

PART A : PHYSICS

Single Choice Type

This section contains **20 Single choice questions**. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. Which of the following statements are correct?

(A) Electric monopoles do not exist whereas magnetic monopoles exist.

(B) Magnetic field lines due to a solenoid at its ends and outside cannot be completely straight and confined.

(C) Magnetic field lines are completely confined within a toroid.

(D) Magnetic field lines inside a bar magnet are not parallel.

(E) $\chi = -1$ is the condition for a perfect diamagnetic material, where χ is its magnetic susceptibility.

Choose the correct answer from the options given below :

(1) (C) and (E) only (2) (B) and (D) only (3) (A) and (B) only (4) (B) and (C) only

Ans. 1

Sol. **Statement (C)** is correct because, the magnetic field outside the toroid is zero and they form closed loops inside the toroid itself.

Statement (E) is correct because we know that super conductors are materials inside which the net magnetic field is always zero and they are perfect diamagnetic.

$\mu_r = 1 + \chi$

$\chi = -1$

$\mu_r = 0$

For superconductors.

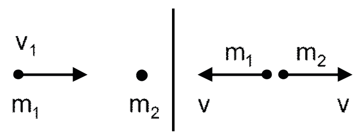
2. An object of mass m_1 collides with another object of mass m_2, which is at rest. After the collision the objects move with equal speeds in opposite direction. The ratio of the masses $m_2 : m_1$ is :

(1) 3 : 1 (2) 2 : 1 (3) 1 : 2 (4) 1 : 1

Ans. 1

Sol. $m_1v_1 = -m_1v + m_2v$

$$v_1 = -v + \frac{m_2}{m_1}v$$



$$\frac{(v_1 + v)}{v} = \frac{m_2}{m_1}$$

$$e = \frac{2v}{v_1} = 1$$

$$v = \frac{v_1}{2}$$

$$\frac{v_1 + v_1/2}{v_1/2} = \frac{m_2}{m_1}$$

$$3 = \frac{m_2}{m_1}$$

3. For an adiabatic expansion of an ideal gas, the fractional change in its pressure is equal to (where γ is the ratio of specific heats)::-

(1) $-\gamma\frac{dV}{V}$ (2) $-\gamma\frac{V}{dV}$ (3) $-\frac{1}{\gamma}\frac{dV}{V}$ (4) $\frac{dV}{V}$

Ans. 1

Sol. PVγ = constant

differentiating

$$\frac{dP}{dV} = -\frac{\gamma P}{V}$$

$$\frac{dP}{P} = -\frac{\gamma dV}{V}$$

4. A proton and an α-particle, having kinetic energies K_p and K_α, respectively, enter into a magnetic field at right angles.

The ratio of the radii of trajectory of proton to that of α-particle is 2 : 1. The ratio of $K_p : K_\alpha$ is :

(1)1 : 8 (2) 8 : 1 (3) 1 : 4 (4) 4 : 1

Ans. 4

Sol. $r = \frac{mv}{qB} = \frac{p}{qB}$ $\frac{m_\alpha}{m_p} = 4$

$$\frac{r_p}{r_\alpha} = \frac{p_p}{q_p}\frac{q_\alpha}{p_\alpha} = \frac{2}{1}$$

$$\frac{p_p}{p_\alpha} = \frac{2q_p}{q_\alpha} = 2\left(\frac{1}{2}\right)$$

$$\frac{p_p}{p_\alpha} = 1$$

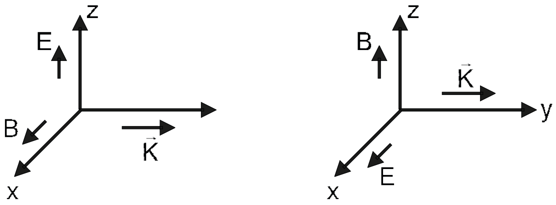
$$\frac{K_p}{K_\alpha} = \frac{p_p^2}{p_\alpha^p}\frac{m_\alpha}{m_p} = (1)\,(4)$$

5. A plane electromagnetic wave propagating along y-direction can have the following pair of electric field $(\vec{E})$ and magnetic field $(\vec{B})$ components.

(1) E_y, B_y or E_z, B_z (2) E_y, B_x or E_x, B_y (3) E_x, B_z or E_z, B_x (4) E_x, B_y or E_y, B_x

Ans. 3

Sol.



6. Consider a uniform wire of mass M and length L. It is bent into a semicircle. Its moment of inertia about a line perpendicular to the plane of the wire passing through the centre is ::-

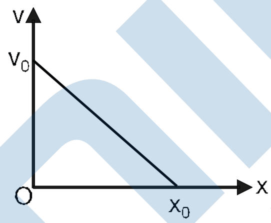
(1) $\frac{1}{4}\frac{ML^2}{\pi^2}$ (2) $\frac{2}{5}\frac{ML^2}{\pi^2}$ (3) $\frac{ML^2}{\pi^2}$ (4) $\frac{1}{2}\frac{ML^2}{\pi^2}$

Ans. 3

Sol. $\pi r = L \Rightarrow r = \frac{L}{\pi}$

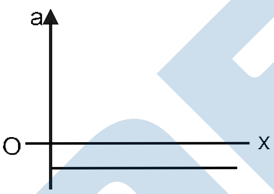
$I = Mr^2 = \frac{ML^2}{\pi^2}$

7. The velocity-displacement graph of a particle is shown in the figure.

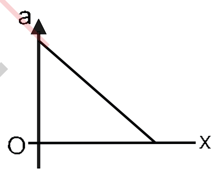


The acceleration-displacement graph of the same particle is represented by :

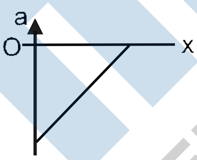
(1)



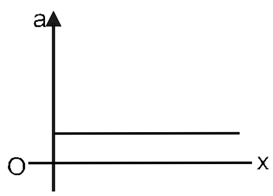
(2)



(3)



(4)



Ans. 3

Sol. $v = \left(\frac{v_0}{x_0}\right)x + v_0$

$a = \frac{vdv}{dx}$

$a = \left[-\left(\frac{v_0}{x_0}\right)x + v_0\right]\left[-\frac{v_0}{x_0}\right]$

$$a = \left(\frac{v_0}{x_0}\right)^2 x - \frac{v_0^2}{x_0}$$

8. The correct relation between α (ratio of collector current to emitter current) and β (ratio of collector current to base current) of a transistor is :

(1) $\beta = \frac{\alpha}{1+\alpha}$ (2) $\alpha = \frac{\beta}{1-\alpha}$ (3) $\beta = \frac{1}{1-\alpha}$ (4) $\alpha = \frac{\beta}{1+\beta}$

Ans. 4

Sol. $\alpha = \frac{I_C}{I_E}, \beta = \frac{I_C}{I_B}$

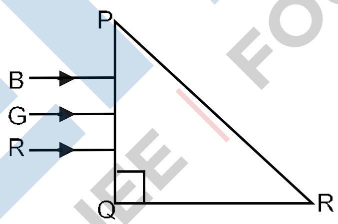
$I_E = I_B + I_C$

$$\alpha = \frac{I_C}{I_B + I_C} = \frac{1}{\frac{I_B}{I_C} + 1}$$

$$\alpha = \frac{1}{\frac{I}{\beta} + 1}$$

$$\alpha = \frac{\beta}{1+\beta}$$

9. Three rays of light, namely red (R), green (G) and blue (B) are incident on the face PQ of a right angled prism PQR as shown in figure.

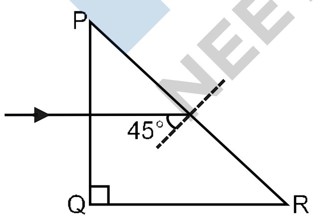


The refractive indices of the material of the prism for red, green and blue wavelength are 1.27, 1.42 and 1.49 respectively. The colour of the ray(s) emerging out of the face PR is :

(1) green (2) red (3) blue and green (4) blue

Ans. 2

Sol.



Assuming that the right angled prism is an isosceles prism, so the other angles will be 45° each.

$\Rightarrow$ Each incident ray will make an angle of 45° with the normal at face PR.

$\Rightarrow$ The wavelength corresponding to which the incidence angle is less than the critical angle, will pass through PR.

$\Rightarrow \theta_C$ = critical angle

$$\Rightarrow \theta_C = \sin^{-1}\left(\frac{1}{\mu}\right)$$

$\Rightarrow$ If $\theta_C \geq 45°$

the light ray will pass

$$\Rightarrow (\theta_C)_{Red} = \sin^{-1}\left(\frac{1}{1.27}\right) = 51.94°$$

Red will pass.

$$\Rightarrow (\theta_C)_{Green} = \sin^{-1}\left(\frac{1}{1.42}\right) = 44.76°$$

Green will not pass

$$\Rightarrow (\theta_C)_{Blue} = \sin^{-1}\left(\frac{1}{1.49}\right) = 42.15°$$

Blue will not pass

$\Rightarrow$ So only red will pass through PR.

10. If the angular velocity of earth's spin is increased such that the bodies at the equator start floating, the duration of the day would be approximately :

(Take : g = 10 ms^{-2}, the radius of earth, R = 6400 × 10^3 m, Take π = 3.14)

(1) 60 minutes (2) does not change (3) 1200 minutes (4) 84 minutes

Ans. 4

Sol. For objects to float

$mg = m\omega^2 R$

ω = angular velocity of earth.

R = Radius of earth

$$\omega = \sqrt{\frac{g}{R}} \quad \text{........(1)}$$

Duration of day = T

$$T = \frac{2\pi}{\omega} \quad \text{........(2)}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{R}{g}}$$

$$= 2\pi\sqrt{\frac{6400 \times 10^3}{10}}$$

$\Rightarrow \frac{T}{60} = 83.775$ minutes

$\approx$ 84 minutes

11. The decay of a proton to neutron is :

(1) not possible as proton mass is less than the neutron mass

(2) possible only inside the nucleus

(3) not possible but neutron to proton conversion is possible

(4) always possible as it is associated only with $\beta+$ decay

Ans. 2

Sol. It is possible only inside the nucleus and not otherwise.

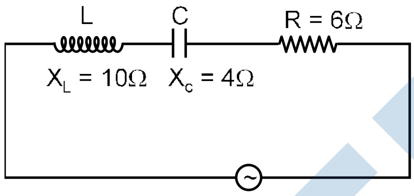
12. In a series LCR circuit, the inductive reactance (X_L) is 10 Ω and the capacitive reactance (X_C) is 4 Ω. The resistance (R) in the circuit is 6 W.

The power factor of the circuit is :

(1) $\frac{1}{2}$ (2) $\frac{1}{2\sqrt{2}}$ (3) $\frac{1}{\sqrt{2}}$ (4) $\frac{\sqrt{3}}{2}$

Ans. 3

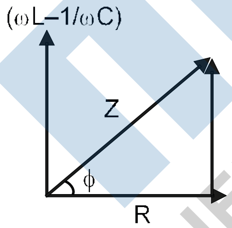
Sol.



We know that power factor is $\cos\phi$.

$\cos\phi = \frac{R}{Z}$ ......(1)

$Z = \sqrt{R^2 + (X_L - X_C)^2}$ ......(2)



$\Rightarrow Z = \sqrt{6^2 + (10-4)^2}$

$\Rightarrow z = 6\sqrt{2} \mid \cos\phi = \frac{6}{6\sqrt{2}}$

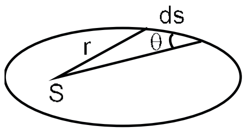
$\cos\phi = \frac{1}{\sqrt{2}}$

13. The angular momentum of a planet of mass M moving around the sun in an elliptical orbit is $\vec{L}$. The magnitude of the areal velocity of the planet is :

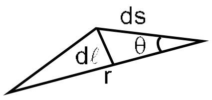
(1) $\frac{4L}{M}$ (2) $\frac{L}{M}$ (3) $\frac{2L}{M}$ (4) $\frac{L}{2M}$

Ans. 4

Sol.



For small displacement ds of the planet its area can be written as



$$dA = \frac{1}{2} r d\ell = \frac{1}{2} r ds \sin\theta$$

$$\text{A. vel} = \frac{dA}{dt} = \frac{1}{2} r \sin\theta \frac{ds}{dt} = \frac{Vr\sin\theta}{2}$$

$$\frac{dA}{dt} = \frac{1}{2}\frac{mVr\sin\theta}{m} = \frac{L}{2m}$$

14. The function of time representing a simple harmonic motion with a period of $\frac{\pi}{\omega}$ is :

(1) sin (ωt) + cos (ωt)
(2) cos (ωt) + cos (2ωt) + cos (3ωt)
(3) $\sin^2(\omega t)$
(4) $3\cos\left(\frac{\pi}{4} - 2\omega t\right)$

Ans. 4

Sol. Time period $T = \frac{2\pi}{\omega'}$

$$\frac{\pi}{\omega} = \frac{2\pi}{\omega'}$$

$\omega' = 2\omega \rightarrow$ Angular frequency of SHM Option (3)

$$\sin^2 \omega t = \frac{1}{2}(2\sin^2 \omega t) = \frac{1}{2}(1 - \cos 2\omega t)$$

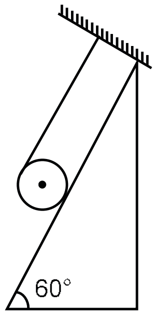
Angular frequency of $\left(\frac{1}{2} - \frac{1}{2}\cos 2\omega t\right)$ is 2ω Option (4)

Angular frequency of SHM

$3\cos\left(\frac{\pi}{4} - 2\omega t\right)$ is2ω.

So option (3) & (4) bot have angular frequency 2ω but option (4) is direct answer.

15. A solid cylinder of mass m is wrapped with an inextensible light string and, is placed on a rough inclined plane as shown in the figure. The frictional force acting between the cylinder and the inclined plane is :

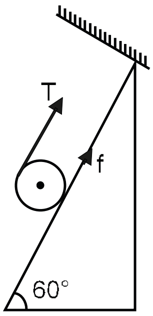


[The coefficient of static friction, μ_s, is 0.4]

(1) $\frac{7}{2}mg$ (2) 5 mg (3) $\frac{mg}{5}$ (4) 0

Ans. 3

Sol.



Let's take solid cylinder is in equilibrium

T + f = mg sin60 ....(i)

TR – fR = 0 ....(ii)

Solving we get

$$T = f_{req} = \frac{mg\sin\theta}{2}$$

But limiting friction < required friction

$$\mu \text{ mg} \cos 60° < \frac{mg \sin 60°}{2}$$

∴ Hence cylinder will not remain in equilibrium

Hence f = kinetic

$= \mu_k N$

$= \mu_k$ mg cos 60°

$$= \frac{mg}{5}$$

16. The time taken for the magnetic energy to reach 25% of its maximum value, when a solenoid of resistance R, inductance L is connected to a battery, is :

(1) $\frac{L}{R}\ell n5$ (2) infinite (3) $\frac{L}{R}\ell n2$ (4) $\frac{L}{R}\ell n10$

Ans. 3

Sol. Magnetic energy $= \frac{1}{2}Li^2 = 25\%$

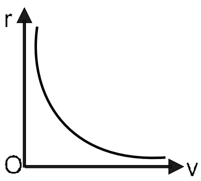
$ME \Rightarrow 25\% \Rightarrow i = \frac{i_0}{2}$

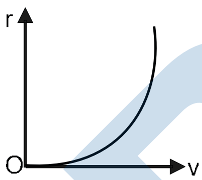
$i = i_0\,(1 – R^{–Rt/L})$ for charging

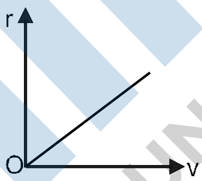
$t = \frac{L}{R}\ell n2$

17. A particle of mass m moves in a circular orbit under the central potential field, $U(r) = \frac{-C}{r}$, where C is a positive constant.

The correct radius – velocity graph of the particle's motion is :

(1)


(2)


(3)


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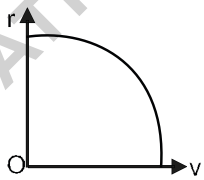


Ans. 1

Sol. $U = -\frac{C}{r}$

$F = -\frac{dU}{dr} = -\frac{C}{r^2}$

$|F| = \frac{mv^2}{r}$

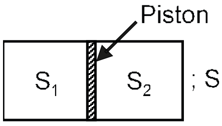
$\frac{C}{r^2} = \frac{mv^2}{r}$

$v^2 \propto \frac{1}{r}$

18. An ideal gas in a cylinder is separated by a piston in such a way that the entropy of one part is S_1 and that of the other part is S_2. Given that $S_1 > S_2$. If the piston is removed then the total entropy of the system will be :

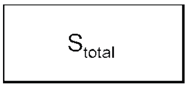
(1) $S_1 \times S_2$ (2) $S_1 - S_2$ (3) $\frac{S_1}{S_2}$ (4) $S_1 + S_2$

Ans. 4

Sol. 

; $S_1 > S_2$

After piston is removed



; $S_{total} = S_1 + S_2$

19. Consider a sample of oxygen behaving like an ideal gas. At 300 K, the ratio of root mean square (rms) velocity to the average velocity of gas molecule would be :

(Molecular weight of oxygen is 32 g/mol; R = 8.3 J K^{-1} mol^{-1})

(1) $\sqrt{\frac{3}{3}}$ (2) $\sqrt{\frac{8}{3}}$ (3) $\sqrt{\frac{3\pi}{8}}$ (4) $\sqrt{\frac{8\pi}{3}}$

Ans. 3

Sol. $v_{rms} = \sqrt{\frac{3RT}{M}}$

$v_{avg} = \sqrt{\frac{8}{\pi}\frac{RT}{M}}$

$\frac{v_{rms}}{v_{avg}} = \sqrt{\frac{3\pi}{8}}$

20. The speed of electrons in a scanning electron microscope is 1×10^7 ms^{-1}. If the protons having the same speed are used instead of electrons, then the resolving power of scanning proton microscope will be changed by a factor of:

(1) 1837 (2) $\frac{1}{1837}$ (3) $\sqrt{1837}$ (4) $\frac{1}{\sqrt{1837}}$

Ans. 1

Sol. Resolving power (RP) $\propto \frac{1}{\lambda}$

$\lambda = \frac{h}{P} = \frac{h}{mv}$

So (RP) $\propto \frac{mv}{h}$

RP $\propto$ P

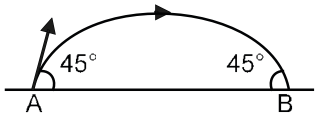
RP $\propto$ mv

RP $\propto$ m

Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

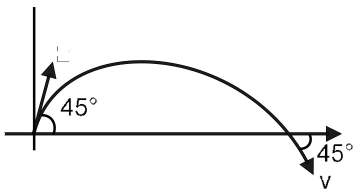
1. The projectile motion of a particle of mass 5 g is shown in the figure.



The initial velocity of the particle is $5\sqrt{2}$ ms^{-1} and the air resistance is assumed to be negligible. The magnitude of the change in momentum between the points A and B is x × 10^{-2} kgms^{-1}. The value of x, to the nearest integer, is _______.

Ans. 5

Sol.



$|\vec{u}| = |\vec{v}|$ ......(1)

$\vec{u} = u\cos 45\,\hat{i} + u\sin 45\,\hat{j}$ ......(2)

$\vec{v} = v\cos 45\,\hat{i} - v\sin 45\,\hat{j}$ ....(3)

$|\overline{\Delta P}| = |m(\vec{v} - \vec{u})|$ ......(4)

$\Delta P = 2mu\sin 45°$

$= 2 \times 5 \times 10^{-3} \times 5\sqrt{2} \times \frac{1}{\sqrt{2}}$

$= 50 \times 10^{-3}$

$= 5 \times 10^{-2}$

2. A ball of mass 4 kg, moving with a velocity of 10 ms^{-1}, collides with a spring of length 8 m and force constant 100 Nm^{-1}. The length of the compressed spring is x m. The value of x, to the nearest integer, is_____.

Ans. 6

Sol. Let's say the compression in the spring by : y.

So, by work energy theorem we have

$\Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}ky^2$

$\Rightarrow y = \sqrt{\frac{m}{k}}.v$

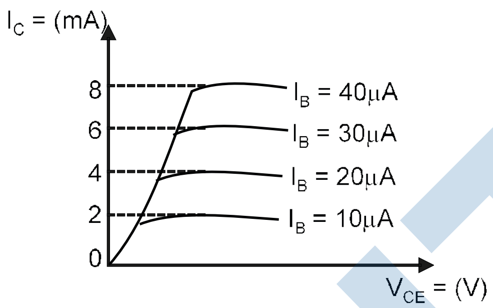
$\Rightarrow y = \sqrt{\frac{4}{100}} \times 10$

$\Rightarrow y = 2m$

$\Rightarrow$ final length of spring

$= 8 - 2 = 6m$

3. The typical output characteristics curve for a transistor working in the common-emitter configuration is shown in the figure.



The estimated current gain from the figure is

Ans. 200

Sol. $\beta = \frac{\Delta I_C}{\Delta I_B} = \frac{2 \times 10^{-3}}{10 \times 10^{-6}}$

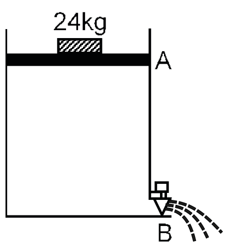
$\beta = \frac{1}{5} \times 10^3$

$\beta = 2 \times 10^2$

$\beta = 200$

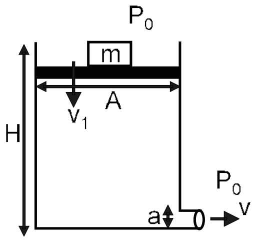
4. Consider a water tank as shown in the figure. It's cross-sectional area is 0.4 m^2. The tank has an opening B near the bottom whose crosssection area is 1 cm^2. A load of 24 kg is applied on the water at the top when the height of the water level is 40 cm above the bottom, the velocity of water coming out the opening B is v ms^{-1}. The value of v, to the nearest integer, is___.

[Take value of g to be 10 ms^{-2}]



Ans. 3

Sol.



m = 24 kg

A = 0.4 m^2

a = 1 cm^2

H = 40cm

Using Bernoulli's equation

$$\Rightarrow \left(P_0 + \frac{mg}{A}\right)\rho gH + \cancelto{0}{\frac{1}{2}\rho v_1^2}$$

$$= P_0 + 0 + \frac{1}{2}\rho v^2 \qquad \text{......(1)}$$

$\Rightarrow$ Neglecting v_1

$$\Rightarrow v = \sqrt{2gH + \frac{2mg}{A\rho}}$$

$$\Rightarrow v = \sqrt{8 + 1.2}$$

$\Rightarrow$ v = 3.033 m/s

$\Rightarrow v \approx 3$ m/s

5. A TV transmission tower antenna is at a height of 20 m. Suppose that the receiving antenna is at.

(i) ground level

(ii) a height of 5 m.

The increase in antenna range in case (ii) relative to case (i) is n%.

The value of n, to the nearest integer, is .

Ans. 50

Sol. $\text{Range} = \sqrt{2Rh}$

$\text{Range (i)} = \sqrt{2Rh}$

$\text{Range (ii)} = \sqrt{2Rh} + \sqrt{2Rh'}$

where h = 20 m & h' = 5m

$$\text{Ans} = \frac{\sqrt{2Rh'}}{\sqrt{2Rh}} \times 100\% = \frac{\sqrt{5}}{\sqrt{20}} \times 100\% = 50\%$$

6. The radius of a sphere is measured to be (7.50 ± 0.85) cm. Suppose the percentage error in its volume is x. The value of x, to the nearest x, is_______.

Ans. 34

Sol. $\because v = \frac{4}{3}\pi r^3$

taking log & then differentiate

$$\frac{dV}{V} = 3\frac{dr}{r}$$

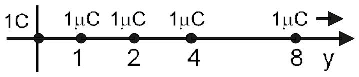
$$= \frac{3 \times 0.85}{7.5} \times 100\% = 34\%$$

7. An infinite number of point charges, each carrying 1 μC charge, are placed along the y-axis at y = 1 m, 2 m, 4 m, 8 m................. The total force on a 1 C point charge, placed at the origin, is x × 10^3 N. The value of x, to the nearest integer, is________.

[Take $\frac{1}{4\pi \in_0}$ = 9 × 10^9 Nm^2/C^2]

Ans. 12

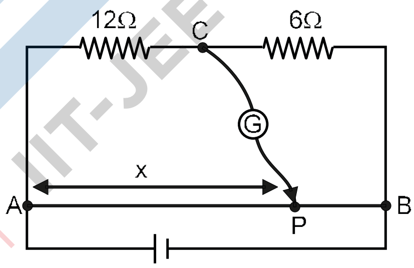
Sol.



$$F = k(1C)(1\mu C)\left[1 + \frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{8^2} + ...\right]$$

$$= 9 \times 10^3 \left[\frac{1}{1 - \frac{1}{4}}\right] = 12 \times 10^3 N$$

8. Consider a 72 cm long wire AB as shown in the figure. The galvanometer jockey is placed at P on AB at a distance x cm from A. The galvanometer shows zero deflection.



The value of x, to the nearest integer, is

Ans. 48

Sol. In Balanced conditions

$$\frac{12}{6} = \frac{x}{72 - x}$$

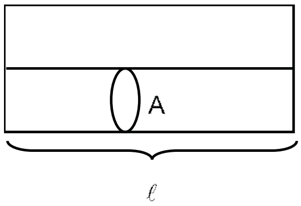
X = 48 cm

9. Two wires of same length and thickness having specific resistances 6Ω cm and 3Ω cm respectively are connected in parallel. The effective resistivity is ρ Ω cm. The value of ρ, to the nearest integer, is_____.

Ans. 4

Sol. ∵ in parallel

$$R_{net} = \frac{R_1R_2}{R_1 + R_2}$$



$$\frac{\rho\ell}{2A} = \frac{\rho_1 \frac{\ell}{A} \times \rho_2 \frac{\ell}{A}}{\rho_1 \frac{\ell}{A} + \rho_2 \frac{\ell}{A}}$$

$$\frac{\rho}{2} = \frac{6 \times 3}{6 + 3} = 2$$

ρ = 4

10. A galaxy is moving away from the earth at a speed of 286 kms^{-1}. The shift in the wavelength of a red line at 630 nm is x × 10^{-10} m. The value of x, to the nearest integer, is_____.

[Take the value of speed of light c, as 3 × 10^8 ms^{-1}]

Ans. 6

Sol. $\frac{\Delta\lambda}{\lambda}c = v$

$$\Delta\lambda = \frac{v}{c} \times \lambda = \frac{286}{3 \times 10^5} \times 630 \times 10^{-9} = 6 \times 10^{-10}$$

PART B : CHEMISTRY

Single Choice Type

This section contains **20 Single choice questions.** Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. The oxidation states of nitrogen in NO, NO_2, N_2O and NO_3^- are in the order of :

(1) $NO_3^- > NO_2 > NO > N_2O$ (2) $NO_2 > NO_3^- > NO > N_2O$

(3) $N_2O > NO_2 > NO > NO_3^-$ (4) $NO > NO_2 > N_2O > NO_3^-$

Ans. 1

Sol. The oxidation states of Nitrogen in following molecules are as follows

$NO_3^- \rightarrow +5$

$NO_2 \rightarrow +4$

$NO \rightarrow +2$

$N_2O \rightarrow +1$

2. In basic medium, H_2O_2 exhibits which of the following reactions ?

(A) $Mn^{2+} \rightarrow Mn^{4+}$

(B) $I_2 \rightarrow I^-$

(C) $PbS \rightarrow PbSO_4$

Choose the most appropriate answer from the options given below:

(1) (A), (C) only (2) (A) only (3) (B) only (4) (A), (B) only

Ans. 4

Sol. In basic medium, oxidising action of H_2O_2. $Mn^{2+} + H_2O_2 \rightarrow Mn^{+4} + 2OH^-$

In basic medium, reducing action of H_2O_2

$I_2 + H_2O_2 + 2OH^- \rightarrow 2I^- + 2H_2O + O_2$

In acidic medium, oxidising action of H_2O_2.

$PbS(s) + 4H_2O_2(aq) \rightarrow PbSO_4(s) + 4H_2O(\ell)$

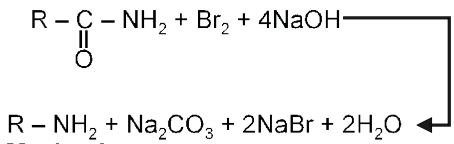
Hence correct option (4)

3. In the reaction of hypobromite with amide, the carbonyl carbon is lost as :

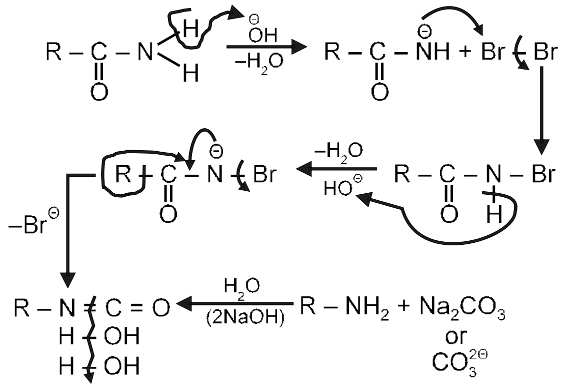
(1) CO_3^{2-} (2) HCO_3^- (3) CO_2 (4) CO

Ans. 1

Sol.



Mechanism



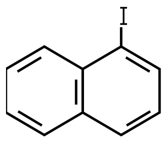
4. The oxide that shows magnetic property is :

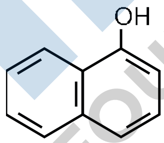
(1) SiO_2 (2) Mn_3O_4 (3) Na_2O (4) MgO

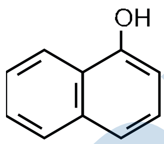
Ans. 2

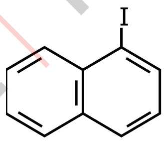
Sol. Mn_3O_4 shows magnetic properties.

5. Main Products formed during a reaction of 1-methoxy naphthalene with hydroiodic acid are:

(1)  and CH_3OH

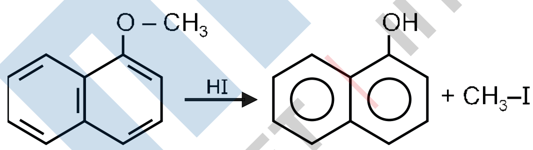
(2)  and CH_3I

(3)  and CH_3OH

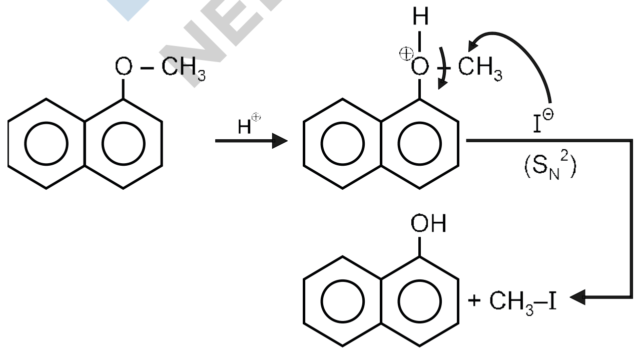
(4)  and CH_3I

Ans. 2

Sol.



Mechanism



6. Deficiency of vitamin K causes :

(1) Increase in blood clotting time
(2) Increase in fragility of RBC's
(3) Cheilosis
(4) Decrease in blood clotting time

Ans. 1

Sol. Due to deficiency of Vitamin K causes increases in blood clotting time.

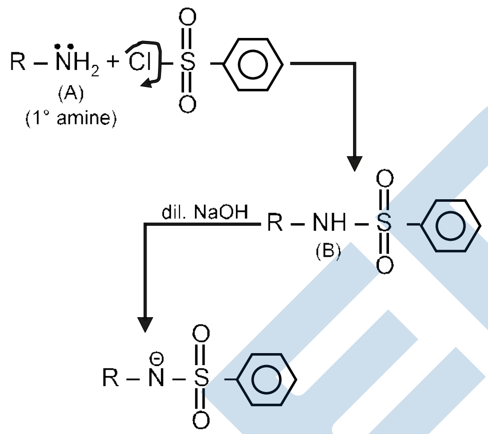
Note : Vitamin K related to blood factor

7. An organic compound "A" on treatment with benzene sulphonyl chloride gives compound B. B is soluble in dil. NaOH solution. Compound A is :

(1) $C_6H_5-N-(CH_3)_2$
(2) $C_6H_5-NHCH_2CH_3$
(3) $C_6H_5-CH_2\ NHCH_3$
(4) $C_6H_5-\underset{\displaystyle CH_3}{\underset{|}{C}H}-NH_2$

Ans. 4

Sol. Hinsberg reagent (Benzene sulphonyl chloride) gives reaction product with 1° amine and it is soluble in dil. NaOH.



8. The first ionization energy of magnesium is smaller as compared to that of elements X and Y, but higher than that of Z. the elements X, Y and Z, respectively, are :

(1) chlorine, lithium and sodium
(2) argon, lithium and sodium
(3) argon, chlorine and sodium
(4) neon, sodium and chlorine

Ans. 3

Sol. The 1^{st} IE order of 3^{rd} period is

Na < Al < Mg < Si < S < P < Cl < Ar

X & Y are Ar & Cl

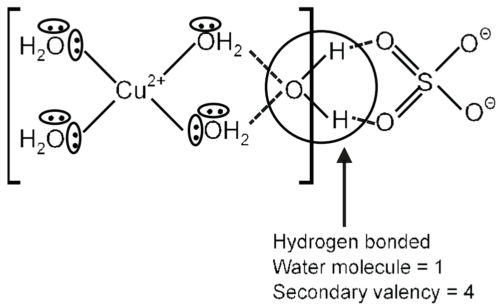
Z is sodium (Na).

9. The secondary valency and the number of hydrogen bonded water molecule(s) in $CuSO_4.5H_2O$, respectively, are :

(1) 6 and 4
(2) 4 and 1
(3) 6 and 5
(4) 5 and 1

Ans. 2

Sol.



10. Given below are two statements :

Statement I : Bohr's theory accounts for the stability and line spectrum of Li^+ ion.

Statement II : Bohr's theory was unable to explain the splitting of spectral lines in the presence of a magnetic field.

In the light of the above statements, choose the most appropriate answer from the options given below :

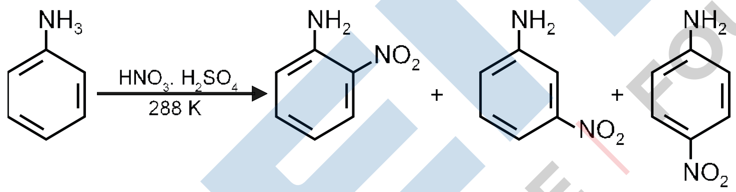
(1) Both statement I and statement II are true.
(2) Statement I is false but statement II is true.
(3) Both statement I and statement II are false.
(4) Statement I is true but statement II is false.

Ans. 2

Sol. Statement-I is false since Bohr's theory accounts for the stability and spectrum of single electronic species (eg : He^+, Li^{2+} etc)

Statement II is true.

11.



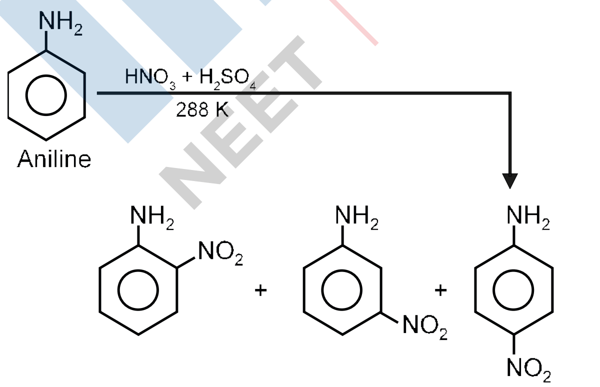
(A) (B) (C)

Consider the given reaction, percentage yield of :

(1) C > A > B (2) B > C > A (3) A > C > B (4) C > B > A

Ans. 4

Sol.



(A) (B) (C)
(2%) (47%) (51%)

% yield order ⇒ C > B > A

12. The charges on the colloidal CdS sol and TiO_2 sol are, respectively :

(1) positive and positive (2) positive and negative

(3) negative and negative (4) negative and positive

Ans. 4

Sol. CdS sol → –ve sol

TiO_2 sol → +ve sol

13. The Match List - I with List - II :

List - I (Class of Chemicals)	List - II (Example)
(a) Antifertility drug	(i) Meprobamate
(b) Antibiotic	(ii) Alitame
(c) Tranquilizer	(iii) Norethindrone
(d) Artificial Sweetener	(iv) Salvarsan

(1) (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i) (2) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)

(3) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii) (4) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

Ans. 3

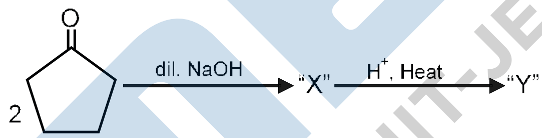
Sol. (A) Antifertility drug → (iii) Norethindrone

(B) Antibiotic → (iv) Salvarsan

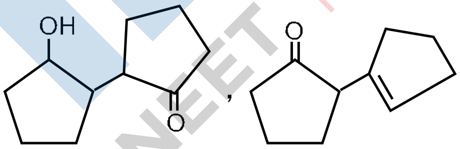
(C) Tranquilizer → (i) Meprobamate

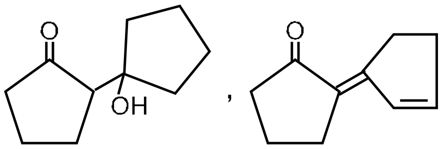
(D) Artificial sweetener → (ii) Alitame

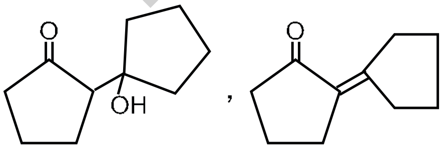
Ans. A–iii, B-iv, C-i, D-ii

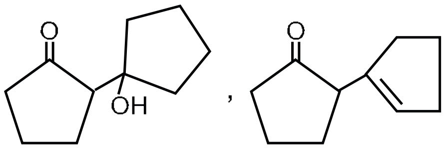
14. 

Consider the above reaction, the product 'X' and 'Y' respectively are :

(1) 

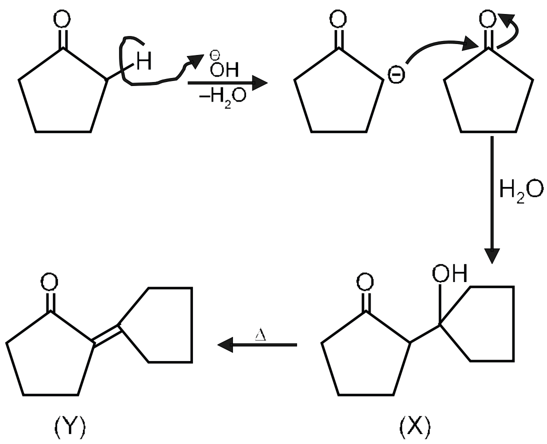
(2) 

(3) 

(4) 

Ans. 3

Sol.



15. Match list-I with list-II :

List-I	**List-II**
(a) Be	(i) Treatment of cancer
(b) Mg	(ii) Extraction of metals
(c) Ca	(iii) Incendiary bombs and signals
(d) Ra	(iv) Windows of X-ray tubes
	(v) Bearings for motor engines.

Choose the most appropriate answer the option given below :

(1) a-iv, b-iii, c-i, d-ii (2) a-iv, b-iii, c-ii, d-i (3) a-iii, b-iv, c-v, d-ii (4) a-iii, b-iv, c-ii, d-v

Ans. 2

Sol. (a) Be $\rightarrow$ it is used in the Windows of X-ray tubes

(b) Mg $\rightarrow$ it is used in the Incendiary bombs and signals

(c) Ca $\rightarrow$ it is used in the Extraction of metals

(d) Ra $\rightarrow$ it is used in the Treatment of cancer

16. Given below are two statements :

Statement I : C_2H_5OH and AgCN both can generate nucleophile.

Statement II : KCN and AgCN both will generate nitrile nucleophile with all reaction conditions.

Choose the most appropriate option :

(1) Statement I is true but statement II is false (2) Both statement I and statement II are true

(3) Statement I is false but statement II is true (4) Both statement I and statement II are false

Ans. 1

Sol. No solution

17. Given below are two statements :

Statement I : Non-biodegradable wastes are generated by the thermal power plants.

Statement II : Bio-degradable detergents leads to eutrophication.

In the light of the above statements, choose the most appropriate answer from the option given below :

(1) Both statement I and statement II are false (2) Statement I is true but statement II is false

(3) Statement I is false but statement II is true (4) Both statement I and statement II are true.

Ans. 4

Sol. Non-biodegradable wastes are generated by the thermal power plants which produces fly ash. Detergents which are biodegradable causes problem called eutrophication which kills animal life by deprieving it of oxygen.

18. Match list-I with list-II :

List-I	List-II
(a) Mercury	(i) Vapour phase refining
(b) Copper	(ii) Distillation refining
(c) Silicon	(iii) Electrolytic refining
(d) Nickel	(iv) Zone refining

Choose the most appropriate answer from the option given below :

(1) a-i, b-iv, c-ii, d-iii (2) a-ii, b-iii, c-i, d-iv (3) a-ii, b-iii, c-iv, d-i (4) a-ii, b-iv, c-iii, d-i

Ans. 3

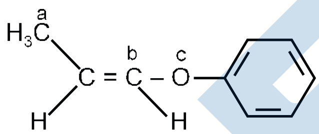
Sol. (a) Mercury $\rightarrow$ Distillation refining

(b) Copper $\rightarrow$ Electrolytic refining

(c) Silicon $\rightarrow$ Zone refining

(d) Nickel $\rightarrow$ Vapour phase refining

19. In the following molecules,

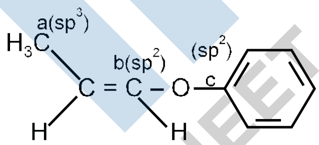


Hybridisation of carbon a, b and c respectively are :

(1) sp^3, sp, sp (2) sp^3, sp^2, sp (3) sp^3, sp^2, sp^2 (4) sp^3, sp, sp^2

Ans. 3

Sol.



20. A hard substance melts at high temperature and is an insulator in both solid and in molten state. This solid is most likely to be a / an :

(1) Ionic solid (2) Molecular solid (3) Metallic solid (4) Covalent solid

Ans. 4

Sol. Covalent or network solid have very high melting point and they are insulators in their solid and molten form.

Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

1. A reaction has a half life of 1 min. The time required for 99.9% completion of the reaction is ______ min.

[Use : ln 2= 0.69, ln 10 = 2.3]

Ans. 10

Sol. $\dfrac{t_{99.9\%}}{t_{50\%}} = \dfrac{\frac{1}{K}\ln\frac{100}{0.1}}{\frac{1}{K}\ln 2}$

$= \dfrac{\ln 1000}{\ln} \times t_{50\%}$ $\qquad = \dfrac{3\ln 10}{\ln 2} \times 1$

$= \dfrac{3 \times 2.3}{0.69} = 10$

2. The molar conductivities at infinite dilution of barium chloride, sulphuric arid and hydrochloric acid are 280, 860 and 426 Scm^2 mol^{-1} respectively. The molar conductivity at infinite dilution of barium sulphate is ________S cm^2 mol^{-1}

Ans. 288

Sol. $\Lambda_m^\infty\ (BaSO_4) = \lambda_m^\infty(Ba^{2+}) + \lambda_m^\infty(SO_4^{2-})$

$\Lambda_m^\infty\ (BaSO_4) = \Lambda_m^\infty(BaCl_2) + \Lambda_m^\infty(H_2SO_4) - 2\ \Lambda_m^\infty\ (HCl)$

= 280 + 860 – 2 (426)

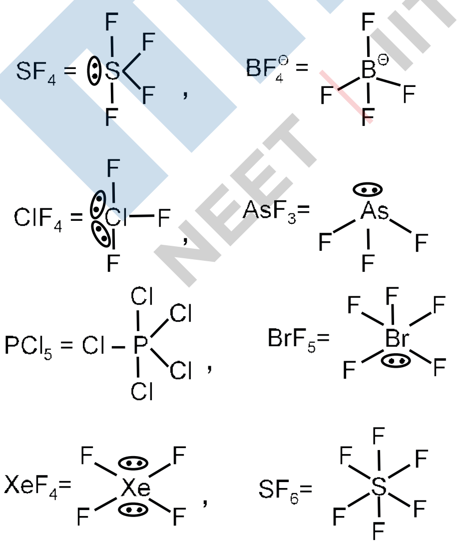
= 288 Scm^2 mol^{-1}

3. The number of species below that have two lone pairs of electrons in their central atom is ____

SF_4, BF_4^- , ClF_3, AsF_3, PCl_5, BrF_5, XeF_4, SF_6

Ans. 2

Sol.



Two l.p. on central atom is = ClF_3, XeF_4

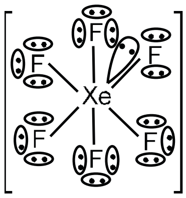
4. A xenon compound 'A' upon partial hydrolysis gives XeO_2F_2. The number of lone pair of electrons present in compound A is _______

Ans. 19

Sol. $XeF_6 + 2H_2O \longrightarrow XeO_2F_2 + 4HF$

(A) (Limited water)

Structure of 'A'



Total l.p. on (A) = 19

5. The gas phase reaction

$2A(g) \rightleftharpoons A_2(g)$

at 400 K has $\Delta G^o = + 25.2$ kJ mol^{-1}.

The equilibrium constant K_C for this reaction is _______ $\times 10^{-2}$.

[Use : R = 8.3 J $mol^{-1}K^{-1}$, ln 10 = 2.3

$\log_{10} 2 = 0.30$, 1 atm = 1 bar]

[antilog (–0.3) = 0.501]

Ans. 166

Zigyan ans. by (2)

Sol. Using formula

$\Delta_r G° = -RT\ln K_p$

$25200 = -2.3 \times 8.3 \times 400 \log(K_p)$

$K_p = 10^{-3.3} = 10^{-3} \times 0.501$

$= 5.01 \times 10^{-4}$ Bar^{-1}

$= 5.01 \times 10^{-9}$ Pa^{-1}

$= \dfrac{K_c}{8.3 \times 400}$

$K_C = 1.66 \times 10^{-5}$ m^3/mole

$= 1.66 \times 10^{-2}$ L/mol

Ans = 2

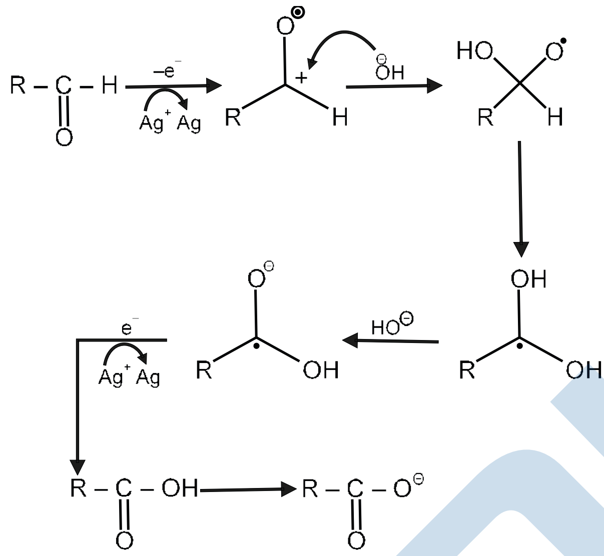
6. In Tollen's test for aldehyde, the overall number of electron(s) transferred to the Tollen's reagent formula $[Ag(NH_3)_2]^+$ per aldehyde group to form silver mirror is_______.

Ans. 2

Sol. $AgNO_3 + NaOH \rightarrow AgOH + NaNO_3$

$2AgOH \rightarrow Ag_2O + H_2O$

$$Ag_2O + 4NH_3 + H_2O \rightarrow 2Ag(NH_3)_2^+ + 2\overset{1}{O}H$$



Total $2e^-$ transfer to Tollen's reagent

7. The solubility of $CdSO_4$ in water is 8.0×10^{-4} mol L^{-1}. Its solubility in 0.01 M H_2SO_4 solution is ______$\times 10^{-6}$ mol L^{-1}.

(Assume that solubility is much less than 0.01 M)

Ans. 64

Sol. In pure water,

$K_{sp} = S^2 = (8 \times 10^{-4})^2$

$= 64 \times 10^{-8}$

In 0.01 M H_2SO_4

$$H_2SO_{4(aq)} \rightarrow \underset{0.02}{2H^+_{(aq)}} + \underset{0.01}{SO_4^{2-}(aq.)}$$

$$\underset{x}{BaSO_{4(s)}} \rightleftharpoons \underset{x}{Ba^{2+}_{(aq.)}} + \underset{(x+0.01)}{SO^{2-}_{4\ (aq.)}}$$

$K_{sp} = x\,(x + 0.01) \quad = 64 \times 10^{-8}$

$x + 0.01 \cong 0.01$ M

So, $x\,(0.01) = 64 \times 10^{-8}$

$x = 64 \times 10^{-6}$ M

8. The A solute a dimerizes in water. The boiling point of a 2 molar solution of A is 100.52ºC. The percentage association of A is.____.

[Use : K_b for water = 0.52 K kg mol^{-1}

Boiling point of water = 100°C]

Ans. 50

Ans. By Zigyan (100)

Sol. $\Delta T_b = T_b - T_b^0$

100.52 – 100

= 0.52°C

$i = \left(1 - \frac{\alpha}{2}\right)$ $\quad\because \Delta T_b = i\, K_b \times m$

$0.52 = \left(1 - \frac{\alpha}{2}\right) \times 0.52 \times 2$

$\alpha = 1$

So, percentage association = 100%

9. 10.0 ml of Na_2CO_3 solution is titrated against 0.2 M HCl solution. The following titre values were obtained in 5 readings.

4.8 ml, 4.9 ml, 5.0 ml, 5.0 ml and 5.0 ml

Based on these readings, and convention of titrimetric estimation of concentration of Na_2CO_3 solution is ________mM.

Ans. 50

Sol. Most precise volume of HCl = 5 ml

at equivalence point

Meq. of Na_2CO_3 = meq. of HCl.

Let molarity of Na_2CO_3

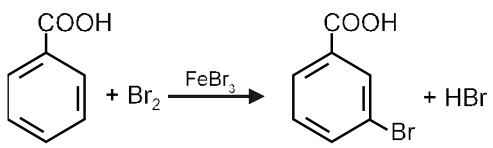
solution = M, then

M × 10 × 2 = 0.2 × 5 × 1

M = 0.05 mol / L

= 0.05 × 1000

= 50 mM

10. 

Consider the above reaction where 6.1 g of benzoic acid is used to get 7.8 g of m-bromo benzoic acid. The percentage yield of the product is_____.

[Given : Atomic masses : C = 12.0u, H : 1.0u, O : 16.0u, Br = 80.0 u]

Ans. 78

Sol. Moles of Benzoic acid = $\frac{6.1}{122}$

= moles of m-bromobenzoic acid

So, weight of m-bromobenzoic acid

$= \frac{6.1}{122} \times 201\,\text{gm}$

$\%\ \text{yield} = \frac{\text{Actual weight}}{\text{Theoretical weight}} \times 100$

$= \frac{7.8}{10.05} \times 100$

= 77.61 %

PART C : MATHEMATICS

Single Choice Type

This section contains **20 Single choice questions**. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. Let y = y (x) be the solution of the differential equation $\frac{dy}{dx} = (y+1)((y+1)e^{x^2/2} - x),\ 0 < x < 2.1$, with y (2) = 0. Then the value of $\frac{dy}{dx}$ at x = 1 is equal to :

(1) $\frac{-e^{3/2}}{(e^2+1)^2}$ (2) $-\frac{2e^2}{(1+e^2)^2}$ (3) $\frac{e^{5/2}}{(1+e^2)^2}$ (4) $\frac{5e^{1/2}}{(e^2+1)^2}$

Ans. 1

Sol. Let y + 1 = Y

$\therefore\ \frac{dY}{dx} = Y^2 e^{\frac{x^2}{2}} - xY$

Put $-\frac{1}{Y} = k$

$\Rightarrow \frac{dk}{dx} + k(-x) = e^{\frac{x^2}{2}}$

$\text{I.F.} = e^{\frac{x^2}{2}}$

$\therefore k = (x+c)e^{x^2/2}$

Put $k = -\frac{1}{y+1}$

$\therefore\ y + 1 = -\frac{1}{(x+c)e^{x^2/2}}$ ....(i)

when x = 2, y = 0, then $c = -2 - \frac{1}{e^2}$

diffentiate equation (i) & put x = 1

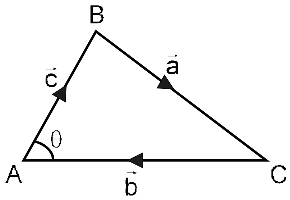
we get $\left(\frac{dy}{dx}\right)_{x=1} = -\frac{e^{3/2}}{(1+e^2)^2}$

2. In a triangle ABC, if $|\overrightarrow{BC}| = 8, |\overrightarrow{CA}| = 7, |\overrightarrow{AB}| = 10$, then the projection of the vector $\overrightarrow{AB}$ on $\overrightarrow{AC}$ is equal to :

(1) $\frac{25}{4}$ (2) $\frac{85}{14}$ (3) $\frac{127}{20}$ (4) $\frac{115}{16}$

Ans. 2

Sol.



$|\vec{a}| = 8, |\vec{b}| = 7, |\vec{c}| = 10$

$\cos\theta = \dfrac{|\vec{b}|^2 + |\vec{c}|^2 - |\vec{a}|^2}{2|\vec{b}||\vec{c}|} = \dfrac{17}{28}$

Projection of $\vec{c}$ on $\vec{b}$

$= |\vec{c}|\cos\theta$

$= 10 \times \dfrac{17}{28}$

$= \dfrac{85}{14}$

3. Let the system of linear equations

$4x + \lambda y + 2z = 0$

$2x - y + z = 0$

$\mu x + 2y + 3z = 0, \lambda, \mu \in R.$

has a non-trivial solution. Then which of the following is true ?

(1) $\mu = 6, \lambda \in R$ (2) $\mu = 2, \lambda \in R$ (3) $\mu = 3, \lambda \in R$ (4) $\mu = -6, \lambda \in R$

Ans. 1

Sol. For non-trivial solution

$$\begin{vmatrix} 4 & \lambda & 2 \\ 2 & -1 & 1 \\ \mu & 2 & 3 \end{vmatrix} = 0$$

$\Rightarrow 2\mu - 6\lambda + \lambda\mu = 12$

when $\mu = 6$, $12 - 6\lambda + 6\lambda = 12$

which is satisfied by all λ

4. Let $f : R - \{3\} \to R - \{1\}$ be defined by $f(x) = \dfrac{x-2}{x-3}$. Let $g : R \to R$ be given as $g(x) = 2x - 3$. Then the sum of all the values of x for which $f^{-1}(x) + g^{-1}(x) = \dfrac{13}{2}$ is equal to

(1)7 (2) 2 (3) 5 (D) 3

Ans. 3

Sol. $f(x) = y = \dfrac{x-2}{x-3}$

$\therefore\ x = \dfrac{3y-2}{y-1}$

$\therefore\ f^{-1}(x) = \dfrac{3x-2}{x-1}$

& $g(x) = y = 2x - 3$

$\therefore\ x = \dfrac{y+3}{2}$

$\therefore\ g^{-1}(x) = \dfrac{x+3}{2}$

$\because\ f^{-1}(x) + g^{-1}(x) = \dfrac{13}{2}$

$\therefore\ x^2 - 5x + 6 = 0 \begin{cases} x_1 \\ x_2 \end{cases}$

$\therefore$ sum of roots

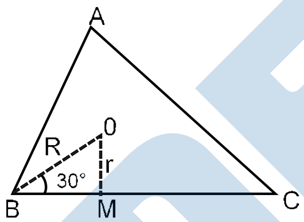
$x_1 + x_2 = 5$

5. Let the centroid of an equilateral triangle ABC be at the origin. Let one of the sides of the equilateral triangle be along the straight line x + y = 3. If R and r be the radius of circumcircle and incircle respectively of ΔABC, then (R + r) is equal to :

(1) $\dfrac{9}{\sqrt{2}}$ (2) $7\sqrt{2}$ (3) $2\sqrt{2}$ (4) $3\sqrt{2}$

Ans. 1

Sol.



$r = OM = \dfrac{3}{\sqrt{2}}$

& $\sin 30° = \dfrac{1}{2} = \dfrac{r}{R} \Rightarrow R = \dfrac{6}{\sqrt{2}}$

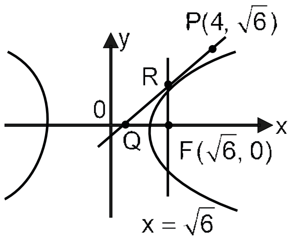
$\therefore\ r + R = \dfrac{9}{\sqrt{2}}$

6. Consider a hyperbola H : $x^2 . 2y^2 = 4$. Let the tangent at a point $P\left(4, \sqrt{6}\right)$ meet the x-axis at Q and latus rectum at $R(x_1, y_1)$, $x_1 > 0$. If F is a focus of H which is nearer to the point P, then the area of ΔQFR is equal to

(1) $4, \sqrt{6}$ (2) $\sqrt{6} - 1$ (3) $\dfrac{7}{\sqrt{6}} - 2$ (4) $4\sqrt{6} - 1$

Ans. 3

Sol.



$$\frac{x^2}{4}-\frac{y^2}{2}=1$$

$$e=\sqrt{1+\frac{b^2}{a^2}}=\sqrt{\frac{3}{2}}$$

$\therefore$ Focus $F(ae, 0) \Rightarrow F(\sqrt{6}, 0)$

Equation of tangent at P to the hyperbola is

$2x-y\sqrt{6}=2$

tangent meet x-axis at Q(1, 0)

& latus rectum $x=\sqrt{6}$ at $R\left(\sqrt{6}, \frac{2}{\sqrt{6}}(\sqrt{6}-1)\right)$

$\therefore$ Area of $\Delta_{QFR}=\frac{1}{2}(\sqrt{6}-1), \frac{2}{\sqrt{6}}(\sqrt{6}-1)$

$=\frac{7}{\sqrt{6}}-2$

7. If P and Q are two statements, then which of the following compound statement is a tautology ?

(1) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow Q$
(2) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow \sim P$
(3) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow P$
(4) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow (P \wedge Q)$

Ans. 2

Sol. LHS of all the options are some i.e.

$((P \rightarrow Q) \wedge \sim Q)$

$\equiv (\sim P \vee Q) \wedge \sim Q$

$\equiv (\sim P \wedge \sim Q) \vee (Q \wedge \sim Q)$

$\equiv \sim P \wedge \sim Q$

(A) $(\sim P \wedge \sim Q) \rightarrow Q$

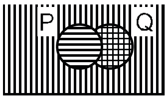
$\equiv \sim(\sim P \wedge \sim Q) \vee Q$

$\equiv (P \vee Q) \vee Q \neq$ tautology

(B) $(\sim P \wedge \sim Q) \rightarrow \sim P$

$\equiv \sim(\sim P \wedge \sim Q) \vee \sim P$

$\equiv (P \vee Q) \vee \sim P$



$\Rightarrow$ Tautology

(C) $(\sim P \wedge \sim Q) \rightarrow P$

$\equiv (P \vee Q) \vee P \neq$ Tautology

(D) $(\sim P \wedge \sim Q) \rightarrow (P \wedge Q)$

$\equiv (P \vee Q) \vee (P \wedge Q) \neq$ Tautology

Aliter :

P	Q	P∨Q	P∨Q	□ P	(P∨Q)∨□ P
T	T	T	T	F	T
T	F	T	F	F	T
F	T	T	F	T	T
F	F	F	F	T	T

8. Let $g(x) = \int_0^x f(t)dt$, where f is continuous function in [0, 3] such that $\frac{1}{3} \le f(t) \le 1$ for all $t \in [0, 1]$ and $0 \le f(t) \le \frac{1}{2}$ for all $t \in (1, 3]$.

The largest possible interval in which g(3) lies is :

(1) $\left[-1, -\frac{1}{2}\right]$ (2) $\left[-\frac{3}{2}, -1\right]$ (3) $\left[\frac{1}{3}, 2\right]$ (4) [1, 3]

Ans. 3

Sol. $\frac{1}{3} \le f(t) \le 1 \;\forall\; t \in [0, 1]$

$0 \le f(t) \le \frac{1}{2} \;\forall\; t \in (1, 3]$

Now, $g(3) = \int_0^3 f(t)dt = \int_0^1 f(t)dt + \int_1^3 f(t)$

$\because \int_0^3 \frac{1}{3}dt \le \int_0^1 f(t)dt \le \int_0^1 1.\,dt$ ....(1)

and $\int_1^3 0\,dt \le \int_1^3 f(1)dt \le \int_1^3 \frac{1}{2}dt$ ......(2)

Adding, we get

$\frac{1}{3} + 0 \le g(3) \le 1 + \frac{1}{2}(3-1)$

$\frac{1}{3} \le g(3) \le 2$

9. Let S_1 be the sum of first 2n terms of an arithmetic progression. Let S_2 be the sum of first 4n terms of the same arithmetic progression. If $(S_2 – S_1)$ is 1000, then the sum of the first 6n terms of the arithmetic progression is equal to:

(1) 1000 (2) 7000 (3) 5000 (4) 3000

Ans. 4

Sol. $S_{2n} = \frac{2n}{2}[2a+(2n-1)d],\ S_{4n} = \frac{4n}{2}[2a+(4n-1)d]$

$\Rightarrow S_2 - S_1 = \frac{4n}{2}[2a+(4n-1)d] - \frac{2n}{2}[2a+(2n-1)d]$

= 4an + (4n – 1)2nd – 2na – (2n – 1)dn

= 2na + nd [8n – 2 – 2n + 1]

$\Rightarrow$ 2na + 2n[6n – 1] = 1000

$2a+(6n-1)d = \frac{1000}{n}$

Now, $S_{6n} = \frac{6n}{2}[2a+(6n-1)d]$

$= 3n.\frac{1000}{n} = 3000$

10. Let a complex number be $w = 1-\sqrt{3}\,i$. Let another complex number z be such that |zw| = 1 and arg(z) – arg(w) = $\frac{\pi}{2}$. Then the area of the triangle with vertices origin, z and w is equal to :

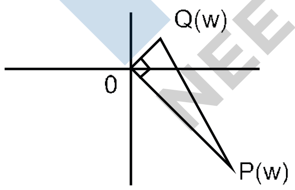
(1) 4 (2) $\frac{1}{2}$ (3) $\frac{1}{4}$ (4) 2

Ans. (2)

Sol. $w = 1-\sqrt{3}\,i \Rightarrow |w| = 2$

Now, $|z| = \frac{1}{|w|} \Rightarrow |z| = \frac{1}{2}$

and $amp(z) = \frac{\pi}{2} + amp(w)$



$\Rightarrow$ Area of triangle $= \frac{1}{2}.OP.OQ$

$= \frac{1}{2}.2.\frac{1}{2} = \frac{1}{2}$

11. Let in a series of 2n observations, half of them are equal to a and remaining half are equal to –a. Also by adding a constant b in each of these observations, the mean and standard deviation of new set become 5 and 20, respectively. Then the value of $a^2 + b^2$ is equal to :

(1) 425 (2) 650 (3) 250 (4) 925

Ans. 1

Sol. Let observations are denoted buy xi for $1 \le i < 2n$

$$\bar{x} = \frac{\sum x_i}{2n} = \frac{(a+a+...+a)-(a+a+...+a)}{2n} \Rightarrow \bar{x} = 0$$

and $\sigma_x^2 = \frac{\sum x_i^2}{2n} - (\bar{x})^2 = \frac{a^2+a^2+...+a^2}{2n} - 0 = a^2 \Rightarrow \sigma_x = a$

Now, adding a constant b then $\bar{y} = \bar{x} + b = 5 \Rightarrow b = 5$

and $\sigma_y = \sigma_x$ (No change in S.D.) $\Rightarrow a = 20 \Rightarrow a^2 + b^2 = 425$

12. Let $S_1 : x^2 + y^2 = 9$ and $S_2 : (x . 2)^2 + y^2 = 1$. Then the locus of center of a variable circle S which touches S_1 internally and S_2 externally always passes through the points :

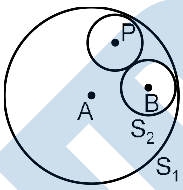
(1) $(0, \pm\sqrt{3})$ (2) $\left(\frac{1}{2}, \pm\frac{\sqrt{5}}{2}\right)$ (3) $\left(2, \pm\frac{3}{2}\right)$ (4) $(1, \pm 2)$

Ans. 3

Sol. $S_1 : x^2 + y^2 = 9$ → $r_1 = 3$, A (0, 0)

$S_2 : (x-2)^2 + y^2 = 1$ → $r_2 = 3$, B (2, 0)

$\because c_1c_2 = r_1 - r_2$



$\therefore$ given circle are touching internally

Let a variable circle with centre P and radius r

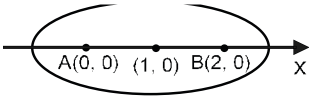
$\Rightarrow PA = r_1 - r$ and $PB = r_2 + r$

$\Rightarrow PA + PB = r_1 + r_2$

$\Rightarrow PA + PB = 4$ (> AB)

$\Rightarrow$ Locus of P is an ellipse with foci at A(0, 0) and B(2, 0) and length of major axis is 2a = 4, $e = \frac{1}{2}$

$\Rightarrow$ centre is at (1. 0) and $b^2 = a^2 (1 - e^2) = 3$ if x-ellipse



$\Rightarrow E : \dfrac{(x-1)^2}{4} + \dfrac{y^2}{3} = 1$

which is satisfied by $\left(2, \pm\dfrac{3}{2}\right)$

13. Let $\vec{a}$ and $\vec{b}$ be two non-zero vectors perpendicular to each other and $|\vec{a}| = |\vec{b}|$. If $|\vec{a} \times \vec{b}| = |\vec{a}|$, then the angle between the vectors $\left(\vec{a} + \vec{b} + \left(\vec{a} \times \vec{b}\right)\right)$ and $\vec{a}$ is equal to :

(1) $\sin^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$ (2) $\cos^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$ (3) $\cos^{-1}\left(\dfrac{1}{\sqrt{2}}\right)$ (4) $\sin^{-1}\left(\dfrac{1}{\sqrt{6}}\right)$

Ans. 2

Sol. $|\vec{a}| = |\vec{b}|, |\vec{a} \times \vec{b}| = |\vec{a}|, \vec{a} \perp \vec{b}$

$|\vec{a} \times \vec{b}| = |\vec{a}| \Rightarrow |\vec{a}||\vec{b}| \sin 90° = |\vec{a}| \Rightarrow |\vec{b}| = 1 = |\vec{a}|$

$\vec{a}$ and $\vec{b}$ are mutually perpendicular unit vectors.

Let $\vec{a} = \hat{i},\ \vec{b} = \hat{j} \Rightarrow \vec{a} \times \vec{b} = \hat{k}$

$\cos\theta = \dfrac{(\hat{i} + \hat{j} + \hat{k}).\hat{i}}{\sqrt{3}\ \sqrt{1}} = \dfrac{1}{\sqrt{3}} \Rightarrow \theta = \cos^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$

14. Let in a Binomial distribution, consisting of 5 independent trials, probabilities of exactly 1 and 2 successes be 0.4096 and 0.2048 respectively. Then the probability of getting exactly 3 successes is equal to :

(1) $\dfrac{32}{625}$ (2) $\dfrac{80}{243}$ (3) $\dfrac{40}{243}$ (4) $\dfrac{128}{625}$

Ans. (1)

Sol. $P(X = 1) = {}^5C_1.p.q^4 = 0.4096$

$P(X = 1) = {}^5C_2.p^2.q^3 = 0.2048$

$\Rightarrow \dfrac{q}{2p} = 2$

$\Rightarrow q = 4p$ and $p + q = 1$

$\Rightarrow p = \dfrac{1}{5}$ and $q = \dfrac{4}{5}$

Now

$P(X = 3) = {}^5C_3.\left(\dfrac{1}{5}\right)^3.\left(\dfrac{4}{5}\right)^2 = \dfrac{10 \times 16}{125 \times 125} = \dfrac{32}{625}$

15. Let a tangent be drawn to the ellipse $\frac{x^2}{27}+y^2=1$ at $(3\sqrt{3}\cos\theta, \sin\theta)$ where $\theta \in \left(0, \frac{\pi}{2}\right)$. Then the value of θ such that the sum of intercepts on axes made by this tangent is minimum is equal to :

(1) $\frac{\pi}{8}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{3}$

Ans. 3

Sol. Equation of tangent be

$$\frac{x\cos\theta}{3\sqrt{3}}+\frac{y.\sin\theta}{1}=1, \qquad \theta \in \left(0, \frac{\pi}{2}\right)$$

Intercept on x-axis

$OA = 3\sqrt{3}\sec\theta$

Intercept on y-axis

$OB = \text{cosec}\,\theta$

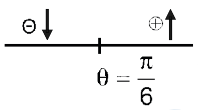
Now, sum of intercept

$= 3\sqrt{3}\sec\theta + \text{cosec}\,\theta = f(\theta)$ let

$f'(\theta) = 3\sqrt{3}\sec\theta\tan\theta - \text{cosec}\,\theta\cot\theta$

$$= 3\sqrt{3}\frac{\sin\theta}{\cos^2\theta}-\frac{\cos\theta}{\sin^2\theta}$$

$$= \underbrace{\frac{\cos\theta}{\sin^2\theta}.3\sqrt{3}}_{\oplus}\left[\tan^2\theta-\frac{1}{3\sqrt{3}}\right]=0 \Rightarrow \theta=\frac{\pi}{6}$$



$\Rightarrow$ at $\theta=\frac{\pi}{6}$, $f(\theta)$ is minimum

16. Define a relation R over a class of n × n real matrices A and B as "ARB iff there exists a non-singular matrix P such that $PAP^{-1} = B$". Then which of the following is true ?

(1) R is symmetric, transitive but not reflexive, (2) R is reflexive, symmetric but not transitive

(3) R is an equivalence relation (4) R is reflexive, transitive but not symmetric

Ans. (3)

Sol. A and B are matrices of n × n order & ARB iff there exists a non singular matrix $P(\det(P) \neq 0)$ such that $PAP^{-1} = B$

For reflexive

$ARA \Rightarrow PAP^{-1} = A$ ...(1) must be true

for P = I, Eq.(1) is true so 'R' is reflexive

For symmetric

$ARB \Leftrightarrow PAP^{-1} = B$ ...(1) is true

for BRA iff PBP.1 = A ...(2) must be true

$\because PAP^{-1} = B$

$P^{-1}PAP^{-1} = P^{-1}B$

$IAP^{-1}P = P^{-1}BP$

$A = P^{-1}BP$ ...(3)

from (2) & (3) $PBP^{-1} = P^{-1}BP$

can be true some $P = P^{-1} \Rightarrow P^2 = I$ $(\det(P) \neq 0)$

So 'R' is symmetric

For trnasitive

$ARB \Leftrightarrow PAP^{-1} = B$... is true

$BRC \Leftrightarrow PBP^{-1} = C$... is true

now $PPAP^{-1}P^{-1} = C$

$P^2A(P^2)^{-1} = C \Rightarrow ARC$

So 'R' is transitive relation

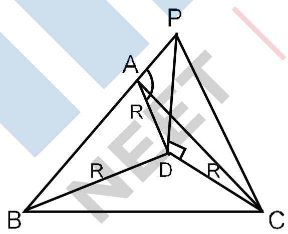
$\Rightarrow$ Hence R is equivalence

17. A pole stands vertically inside a triangular park ABC. Let the angle of elevation of the top of the pole from each corner of the park be $\frac{\pi}{3}$. If the radius of the circumcircle ot ΔABC is 2, then the height of the pole is equal to :

(1) $\frac{2\sqrt{3}}{3}$ (2) $2\sqrt{3}$ (3) $\sqrt{3}$ (4) $\frac{1}{\sqrt{3}}$

Ans. 2

Sol. Let PD = h, R = 2 As angle of elevation of top of pole from A, B, C are equal So D must be circumcentre of ΔABC



$$\tan\left(\frac{\pi}{3}\right) = \frac{PD}{R} = \frac{h}{R}$$

$$h = R\tan\left(\frac{\pi}{3}\right) = 2\sqrt{3}$$

18. If $15\sin^4\alpha + 10\cos^4\alpha = 6$, for some a $\in$ R, then the value of $27\sec^6\alpha + 8\operatorname{cosec}^6\alpha$ is equal to :

(1) 350 (2) 500 (3) 400 (4) 250

Ans. (4)

Sol. $15\sin^4\alpha + 10\cos^4\alpha = 6$

$15\sin^4\alpha + 10\cos^4\alpha = 6(\sin^2\alpha + \cos^2\alpha)^2$

$(3\sin^2\alpha - 2\cos^2\alpha)^2 = 0$

$\tan^2\alpha = \frac{2}{3}, \cot^2\alpha = \frac{3}{2}$

$\Rightarrow 27\sec^6\alpha + 8\operatorname{cosec}^6\alpha$

$= 27(\sec^6\alpha)^3 + 8(\operatorname{cosed}^6\alpha)^3$

$= 27\,(1 + \tan^2\alpha)^3 + 8(1 + \cot^2\alpha)^3$

$= 250$

19. The area bounded by the curve $4y^2 = x^2\,(4 - x)(x - 2)$ is equal to :

(1) $\frac{\pi}{8}$ (2) $\frac{3\pi}{8}$ (3) $\frac{3\pi}{2}$ (4) $\frac{\pi}{16}$

Ans. 3

Sol. $4y^2 = x^2(4 - x)(x - 2)$

$|y| = \frac{|x|}{2}\sqrt{(4-x)(x-2)}$

$\Rightarrow y_1 = \frac{x}{2}\sqrt{(4-x)(x-2)}$

and $y_2 = \frac{-x}{2}\sqrt{(4-x)(x-2)}$

D : $x \in [2, 4]$

Required Area

$= \int_2^4 (y_1 - y_2)dx = \int_2^4 x\sqrt{(4-x)(x-2)}\,dx$ ......(1)

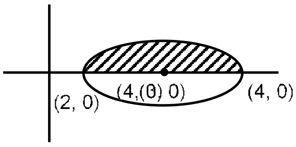
Applying $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

Area $= \int_2^4 (6-x)\sqrt{(4-x)(x-2)}\,dx$ ......(2)

(1) + (2)

$2A = 6\int_2^4 \sqrt{(4-x)(x-2)}\,dx$

$A = 3\int_2^4 \sqrt{1-(x-3)^2}\,dx$



$A = 3.\frac{\pi}{2}.1^2 = \frac{3\pi}{2}$

20. Let $f : R \to R$ be a function defined as

$$f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin 2x}{2x} & , \text{if } x < 0 \\ b & , \text{if } x = 0 \\ \dfrac{\sqrt{x+bx^3} - \sqrt{x}}{bx^{5/2}} & , \text{if } x > 0 \end{cases}$$

If f is continuous at x = 0, then the value of a + b is equal to :

(1) $-\frac{5}{2}$ (2) -2 (3) -3 (4) $-\frac{3}{2}$

Ans. 4

Sol. f(x) is continuous at x = 0

$\lim_{x\to 0^+} f(x) = f(0) = \lim_{x\to 0^-} f(x)$ ......(1)

$f(0) = b$ .....(2)

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} \left(\frac{\sin(a+1)x}{2x} + \frac{\sin 2x}{2x}\right)$

$= \frac{a+1}{2} + 1$ ....(3)

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{\sqrt{x+bx^3} - \sqrt{x}}{bx^{5/2}}$

$\lim_{x\to 0^+} \frac{\sqrt{x}}{\sqrt{x}(\sqrt{1+bx^2}+1)} = \frac{1}{2}$ ....(4)

Use (2), (3) & (4) in (1)

$\frac{1}{2} = b = \frac{a+1}{2} + 1$

$\Rightarrow b = \frac{1}{2}, a = -2$

$a + b = \frac{-3}{2}$

Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

1. If f(x) and g(x) are two polynomials such that the polynomial $P(x) = f(x^3) + xg(x^3)$ is divisible by $x^2 + x + 1$, then P(l) is equal to______.

Ans. 0

Sol. $P(x) = f(x^3) + xg(x^3)$

$P(1) = f(1) + g(1)$ ...(1)

Now P(x) is divisible by $x^2 + x + 1$

$\Rightarrow P(x) = Q(x)(x^2 + x + 1)$

$P(w) = 0 = P(w^2)$ where w, w^2 are non-real cube roots of units

$P(x) = f(x^3) + xg(x^3)$

$P(w) = f(w^3) + wg(w^3) = 0$

$f(1) + wg(1) = 2$ ...(2)

$P(w^2) = f(w^6) + w^2g(w^6) = 0$

$f(1) + w^2g(1) = 0$ ...(3)

(2) + (3)

$\Rightarrow 2f(1) + (w + w^2)g(1) = 0$

$2f(1) = g(1)$ ...(4)

(2) – (3)

$\Rightarrow (w - w^2)g(1) = 0$

$g(1) = 0 = f(1)$ from (4)

from (1) $P(1) = f(1) + g(1) = 0$

2. Let I be an identity matrix of order 2 × 2 and $P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$. Then the value of $n \in N$ for which $P^n = 5I - 8P$ is equal to _______.

Ans. 6

Sol. $P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$

$$5I - 8P = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 16 & -8 \\ 40 & -24 \end{bmatrix} = \begin{bmatrix} -11 & 8 \\ -40 & 29 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} -1 & 1 \\ -5 & 4 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 3 & -2 \\ 10 & -7 \end{bmatrix} \Rightarrow P^6 = \begin{bmatrix} -11 & 8 \\ -40 & 29 \end{bmatrix} = P^n$$

$\Rightarrow n = 6$

3. If $\sum_{r=1}^{10} r!(r^3+6r^2+2r+5)=\alpha(11!)$, then the value of α is equal to _______.

Ans. 160

Sol. $\sum_{r=1}^{10} r!\{(r+1)(r+2)(r+3)-9(r+1)+8\}$

$=\sum_{r=1}^{10}\left[\{(r+3)!-(r+1)!\}-8\{(r+1)!-r!\}\right]$

= (13! + 12! – 2! – 3!) – 8(11! – 1)

= (12.13 + 12 – 8).11! – 8 + 8

= (160)(11)!

Hence α = 160

4. The term independent of x in the expansion of $\left[\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right]^{10}$, $x \neq 1$, is equal to ______,

Ans. 210

Sol. $\left((x^{1/3}-1)-\left(\frac{\sqrt{x}+1}{\sqrt{x}}\right)\right)^{10}$

$(x^{1/2}-x^{1/2})^{10}$

$T_{r+1} = {}^{10}C_r(x^{1/3})^{10-r}(-x^{1/2})^r$

$\frac{10-r}{3}-\frac{r}{2}=0 \Rightarrow 20-2r-3r=0$

$\Rightarrow r = 4$

$T_5 = {}^{10}C_4 = \frac{10\times9\times8\times7}{4\times3\times2\times1}=210$

5. Let P(x) be a real polynomial of degree 3 which vanishes at x = –3. Let P(x) have local minima at x = 1, local maxima at x = –1 and $\int_{-1}^{1} p(x)dx = 18$, then the sum of all the coefficients of the polynomial P(x) is equal to _______.

Ans. 8

Sol. Let p'(x) = a(x – 1) (x + 1) = a(x^2 – 1)

$p(x) = a\int(x^2-1)dx + c\,|$

$=a\left(\frac{x^3}{3}-x\right)+c$

Now p(–3) = 0

$\Rightarrow a\left(-\frac{27}{3}+3\right)+c=0$

$\Rightarrow -6a + c = 0$ ......(1)

Now $\int_{-1}^{1}\left(a\left(\frac{x^3}{3} - x\right) + c\right)dx = 18$

$= 2c = 18 \Rightarrow c = 9$ ......(2)

$\Rightarrow$ from (1) & (2) $\Rightarrow -6a + 9 = 0 \Rightarrow a = \frac{3}{2}$

$\Rightarrow p(x) = \frac{3}{2}\left(\frac{x^3}{3} - x\right) + 9$

sum of coefficient

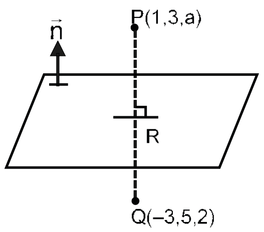
$= \frac{1}{2} - \frac{3}{2} + 9$

= 8

6. Let the mirror image of the point (1, 3, a) with respect to the plane $\vec{r}.(2\hat{i} - \hat{j} + \hat{k}) - b = 0$ be (–3, 5, 2). Then the value of |a + b| is equal to _______.

Ans. 1

Sol.



Plane = 2x – y + z = b

$R = \left(-1, 4, \frac{a+2}{2}\right) \rightarrow$ on plane

$\therefore -2 - 4 + \frac{a+2}{2} = b$

$\Rightarrow$ a + 2 = 2b + 12 $\Rightarrow$ a = 2b + 10 ...(i)

<PQ> = <4, –2, a – 2>

$\therefore \frac{1}{4} = \frac{-1}{-2} = \frac{1}{a-2}$

$\Rightarrow$ a – 2 = 2 $\Rightarrow$ a = 4, b = –3

$\therefore$ |a + b| = 1

7. Let f : R $\rightarrow$ R satisfy the equation f(x + y) = f(x).f(y) for all x, y $\in$ R and f(x) $\neq$ 0 for any x $\in$ R. If lhe function f is differentiable at x = 0 and f'(0) = 3, then $\lim_{x \to 0} \frac{1}{h}(f(h) - 1)$ is equal to ______.

Ans. 3

Sol. If f(x + y) = f(x).f(y) & f'(0) = 3 then

$f(x) = a^x \Rightarrow f'(x) = a^x.\ell na$

$\Rightarrow f'(0) = \ell na = 3 \Rightarrow a = e^3$

$\Rightarrow f(x) = (e^3)^x = e^{3x}$

$$\lim_{x\to 0}\frac{f(x)-1}{x} = \lim_{x\to 0}\left(\frac{e^{3x}-1}{3x}\times 3\right) = 1\times 3 = 3$$

8. Let ${}^nC_r$ denote the binomial coefficient of x^r in the expansion of $(1 + x)^n$.

If $\sum_{k=0}^{10}(2^2+3k)\,{}^nC_k = \alpha.3^{10} + \beta.^{10}$, $\alpha,\beta \in R$, then $\alpha + \beta$ is equal to ______ ,

Ans. 19

Zigyan ans. Bonus

Sol. Instead of ${}^nC_k$ it must be ${}^{10}C_k$ i.e.

$$\sum_{k=0}^{10}(2^2+3k)\,{}^{10}C_k = \alpha.3^{10}+\beta.2^{10}$$

$$LHS = 4\sum_{k=0}^{10}{}^{10}C_k + 3\sum_{k=0}^{10}k.\frac{10}{k}.{}^9C_{k-1}$$

$= 4.2^{10} + 3.10.2^9$

$= 19.2^{10} = \alpha.3^{10} + b.2^{10}$

$\Rightarrow \alpha = 0,\ \beta = 19 \Rightarrow \alpha + \beta = 19$

9. Let P be a plane containing the line $\frac{x-1}{3} = \frac{y+6}{4} = \frac{z+5}{2}$ and parallel to the line $\frac{x-3}{4} = \frac{y-2}{-3} = \frac{z+5}{7}$.

If the point $(1, -1, \alpha)$ lies on the plane P, then the value of $|5\alpha|$ is equal to ______.

Ans. 38

Sol. Equation of plane is $\begin{vmatrix} x-1 & y+6 & z+5 \\ 3 & 4 & 2 \\ 4 & -3 & 7 \end{vmatrix} = 0$

Now $(1, -1, \alpha)$ lies on it so

$$\begin{vmatrix} 0 & 5 & \alpha+5 \\ 3 & 4 & 2 \\ 4 & -3 & 7 \end{vmatrix} = 0 \Rightarrow 5\alpha + 38 = 0 \Rightarrow |5\alpha| = 38$$

10. Let $y = y(x)$ be the solution of the differential equation $xdy - ydx = \sqrt{(x^2-y^2)}\,dx$, $x \geq 1$, with $y(1) = 0$. If the area bounded by the line $x = 1$, $x = e^\pi$, $y = 0$ and $y = y(x)$ is $\alpha e^{2\pi} + \beta$, then the value of $10(\alpha + \beta)$ is equal to ______.

Ans. 4

Sol. $xdy - ydx = \sqrt{(x^2-y^2)}\,dx$

$$\Rightarrow \frac{xdy - ydx}{x^2} = \frac{1}{x}\sqrt{1-\frac{y^2}{x^2}}\,dx$$

$$\Rightarrow \int \frac{d\left(\frac{y}{x}\right)}{\sqrt{1-\left(\frac{y}{x}\right)^2}} = \int \frac{dx}{x}$$

$$\Rightarrow \sin^{-1}\left(\frac{y}{x}\right) = \ell n\,|x| + c$$

at x = 1, y = 0 $\Rightarrow$ c = 0

y = xsin(ℓnx)]

$$A \Rightarrow \int_1^{e^{\pi}} x\sin(\ell nx)dx$$

$$x = e^t, dx = e^t dt \Rightarrow \int_0^{\pi} e^{2t}\sin(t)dt = A$$

$$\alpha e^{2\pi} + \beta = \left(\frac{e^{2t}}{5}(2\sin t - \cos t)\right)_0^{\pi} = \frac{1+e^{2\pi}}{5}$$

$$\alpha = \frac{1}{5}, \beta = \frac{1}{5} \text{ so } 10(\alpha+\beta) = 4$$